GEOMETRY.

293 (Incorrectly numbered 290). Proposed by FRANCIS RUST, C. E., Allegheny, Pa.

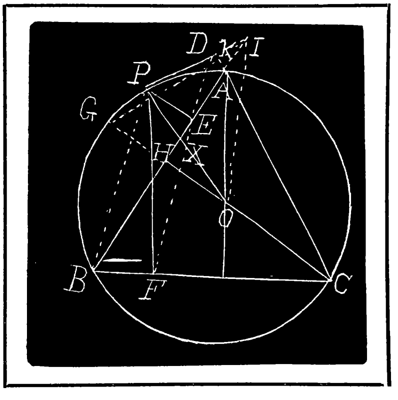
The pedal line of any point on a triangle's circum-circle bisects the distance between this point and the ortho-center of the triangle.

I. Solution by G. W. GREENWOOD, M. A., Dunbar, Pa.

Take, for convenience, an acute triangle ABC and the point P on the arc BC, the feet of the perpendiculars upon BC and CA being D and E, respectively. P is not supposed coincident with B or C. Call the orthocenter O, and *let* BO cut the circle again in G; *let* PQ intersect DE in H and AC in F. Basing the demonstration mainly upon pictorial evidence, we have, since the quadrilateral $PDEC$ is cyclic, $\angle PED=\angle PCD=\angle PGB=\angle GPE$. Hence the triangle PHE is isosceles, and therefore HEF is isosceles, and also $PH=HF$. It can easily be shown that OG is bisected by AC. Hence $\angle OFA=\angle GFA=\angle HFE=\angle HEF$. Hence DE bisects PF and is parallel to OF. It therefore bisects PO.

II. Solution by J. SCHEFFER. A. M., Hagerstown, Md.

Let P be a point in the circumference of the circle, and PD, PE, PF be the perpendiculars let fall from P upon the three sides of $\triangle ABC$, the *straight* line DEF is the pedal for the point P. Let O be the orthocenter, and draw OH perpendicular AB and extend it to G; join G with P and produce it until it meets the pedal DEF at K and the side AB produced at I; draw PB and IO. We now have $\angle PED=\angle PAD=\angle PBC=\angle PGH=\angle IPE$, PE being parallel to GH; hence $\angle KEI=\angle KIE$, being the complements of the equal angles IPE and PED; therefore $PK=KE=IK$. But $\angle HIO=\angle GIH=\angle DEI$. Therefore DX is parallel to IO, X being the point of intersection of PO and the pedal DEF. Since K is the middle point of PI, X must be the middle point of PO. Q. E. D.



294 (Incorrectly numbered 292). Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

Apply the locus of $(x^2+y^2)^3=mx^3$ to the problem of finding a cube m times a given cube.

[No solution has been received.]

295. Proposed by W. J. GREENSTREET, M. A., Editor Mathematical Gazette, Stroud, England.

A variable circle touches an ellipse, and the chord of contact through the other two points of intersection touches a similar coaxial ellipse. Find the locus of the center of the variable circle.